

Web page: <http://www2.physics.umd.edu/~yakovenk/teaching/>

Textbook: Silvio Salinas, *Introduction to Statistical Physics*  
Springer, 2001, ISBN 0-387-95119-9

**Do not forget to write your name and the homework number!**

**Each problem is worth 10 points.**

## Ch. 3 Overview of Classical Thermodynamics

### 1. Problem 3.1, $\mu(P, T)$ .

Among the questions asked in this problem, calculate the *isothermal* compressibility. To solve this problem, just follow the definitions given in the chapter.

### 2. Problem 3.2, Maxwell identities.

To solve this problem, just follow the definitions given in the chapter.

### 3. The entropy change in mixing of two gases.

Consider two *different* gases separated by a wall, as shown in Fig. 3.2. One gas has  $N_1$  particles and occupies the volume  $V_1$ , and another gas has  $N_2$  particles and occupies the volume  $V_2$ . The gases are in thermal equilibrium with each other, so their temperatures are equal.

(a) Suppose the wall is removed, so both gases now occupy the whole volume  $V_1 + V_2$ . Using the expression for the entropy given by Eq. (i) on page 54, which is equivalent to Eq. (3.19), calculate the entropy change  $\Delta S = S_{\text{final}} - S_{\text{initial}}$  in this process. Keep in mind that the temperature remains the same.

Show that the entropy change  $\Delta S > 0$  is always positive in this process.

(b) Suppose the two gases are of the *same* kind. Calculate the entropy change  $\Delta S$  when we remove the wall in this case.

Show that the entropy change is non-negative  $\Delta S \geq 0$ , where the equality is achieved when the initial concentrations in both chambers are equal  $N_1/V_1 = N_2/V_2$ . To show this, study  $\Delta S$  as a function of  $V_1$  for the fixed total volume  $V$  (so that  $V_2 = V - V_1$ ) and the fixed  $N_1$  and  $N_2$  and calculate the first and second derivatives of  $\Delta S$  with respect to  $V_1$  to find the minimum of  $\Delta S$ .

### 4. Adiabatic equations of state.

As discussed in class, the relationships between the pressure  $P$  and the energy density  $u = U/V$  have the following forms for a non-relativistic ideal gas and for a gas of photons or phonons (in 3D):

$$P = \frac{2}{3} u, \quad P = \frac{1}{3} u. \quad (1)$$

(The energy dispersion of elementary excitation in the non-relativistic gas is quadratic  $\varepsilon = p^2/2m$  and in the photon or phonon gas is linear  $\varepsilon = cp$ .)

- (a) Calculate the  $P(V)$  relations for the adiabatic processes in these gases. In the adiabatic process, the system is isolated, and no heat is transferred ( $dS = 0$ ). Thus, you can use the first law of thermodynamics in the form  $dU = -P dV$ . Your answer should have the form  $PV^\alpha = \text{const}$ , where all you need is to calculate the indices  $\alpha$  in both cases.
- (b) We have also derived in the class that  $P \propto u \propto T/V$  for the non-relativistic ideal gas and  $P \propto u \propto T^4$  for a gas of photons or phonons. Substituting these equations into the results from Part (a), find how the temperature  $T$  depends on volume  $V$  when the system is expanded adiabatically. The dependence should have the form  $T = \text{const}/V^\beta$ , where all you need is to calculate the indices  $\beta$  in both cases. Can you give a qualitative explanation of your results by considering quantum-mechanical particles in a box of size  $L$ ? *This part shows how the temperature of the cosmic background radiation decreases when the Universe expands.*

**5. Phonon/photon gas in 2D, inspired by the Qualifier Problem I.3, August 2007.**

Consider an artificial system where phonons or photons can move only in two dimensions (2D) with the constant speed  $c$ . The first law of thermodynamics in this case has the form

$$dU = T dS - \sigma dA, \quad (2)$$

where  $A$  is the area of the system (the 2D analog of  $V$ ), and  $\sigma$  is the surface tension (the 2D analog of  $P$ , equal to the force per unit length).

- (a) Using the arguments similar to those discussed in class, show that

$$U = A f(T) \quad \text{and} \quad \sigma = \frac{1}{2} \frac{U}{A} = \frac{1}{2} f(T), \quad (3)$$

where  $f(T)$  is some (unknown) function of temperature  $T$ .

- (b) Using Eq. (3) rewrite the expression for the entropy

$$dS = \frac{1}{T} dU + \frac{\sigma}{T} dA \quad (4)$$

in terms of  $T$ ,  $A$ , and  $f(T)$ .

By equating the cross-derivatives in Eq. (4), determine the temperature dependence of  $f(T)$  up to a constant factor.

By integrating Eq. (4), determine the temperature and area dependence of the entropy  $S(T, A)$  up to the same constant factor.

- (c) Calculate the temperature dependence of the specific heat at a constant area (the 2D analog of  $c_V$ ):

$$c_A = \frac{1}{A} \left( \frac{\bar{d}Q}{dT} \right)_A. \quad (5)$$