Homework #10 — PHYS 603 — Spring 2008 Professor Victor Yakovenko
Deadline: **Thursday, May 1, 2008, in class** Office: 2115 Physics

Web page: `http://www2.physics.umd.edu/~yakovenk/teaching/`

Textbook: Silvio Salinas, *Introduction to Statistical Physics*
Springer, 2001, ISBN 0-387-95119-9

Do not forget to write your name and the homework number!

Each problem is worth 10 points.

Ch. 11 Phonons and Magnons

1. **Problem 11.1. Debye temperature and Fermi temperature.**

2. **Problem 11.3. $C_P - C_V$ at low temperature.**

   Be sure to do this Problem for C, not c. In this book, the definition is $c = C/N$, which is not very practical for phonons, whose number N is not conserved and depends on temperature.

   To solve this Problem, do the following steps:

   (a) Determine how the entropy S of the 3D phonon gas depends on temperature T at low temperatures. Does S at $T \to 0$ satisfy the Nernst theorem (the third law of thermodynamics)?

   (b) The difference $C_P - C_V$ is given by the formula on page 56, which involves the thermal expansion coefficient α (3.42) and the isothermal compressibility κ_T (3.43). Using the Maxwell relation (3.67), express α in terms of an entropy derivative and determine how α depends on T at low temperatures.

   (c) Argue that the compressibility κ_T (3.43) has a non-zero limit at $T \to 0$, because a crystal has a well-defined rigidity at $T = 0$. Then show that $(C_P - C_V) \propto T^7$ for the 3D phonon gas. Also show that $(C_P - C_V)/C_V \to 0$, so $C_P/C_V \to 1$ at $T \to 0$.

3. **UMD qualifier problem, August 2007: 2D phonons.**

   Consider a square thin film of solid material — only one atom thick — of area $A = L^2$ deposited on an inert substrate. The N atoms may vibrate parallel to the surface, but no perpendicular to it. The speed of sound in this 2D solid is c_S.

   (a) Determine the permitted values of the wave vector $\boldsymbol{q} = (q_x, q_y)$ of the crystal vibrations (phonons) given the size $L \times L$ of the substrate. For simplicity, assume the periodic boundary conditions.

   Write down an equation relating the vibration frequencies ν with the wave vector q and the speed of sound c_S.

   VMY: This problem uses the frequency ν (the number of cycles per second), rather than ω (the angular frequency), which are related as $\nu = \omega/2\pi$.

(b) Show that the density of vibration modes as a function of vibration frequency ν is $D(\nu) = 4\pi A\nu/c_s^2$

VMY: Here the density of states $D(\nu)$ is defined as $d(\#\text{ of modes})/d\nu$. Be sure to include two possible polarizations of a 2D sound wave in the count.

(c) Verify that the maximum vibration frequency $\nu_{\max}$ is

$$\nu_{\max} = \sqrt{\frac{c_s^2 N}{\pi A}}. \qquad (1)$$

VMY: This is the 2D analog of Eq. (11.50).

(d) What is the temperature of the system at which all modes are excitable (the Debye temperature)?

(e) Calculate the average energy $\langle E\rangle$ and the heat capacity at constant area C_A in two limits:

i. High temperature $T \gg h\nu_{\max}$. Note that $e^x \approx 1 + x$ for small x

ii. High temperature $T \ll h\nu_{\max}$. Note that

$$\int_0^\infty \frac{x^2}{e^x - 1}\, dx = 2.404 \qquad (2)$$