Homework #6 — PHYS 603 — Spring 2007 Professor Victor Yakovenko
Deadline: **Tuesday, May 1, 2007, in class** Office: 2115 Physics

Web page: http://www2.physics.umd.edu/˜yakovenk/teaching/

Textbook: Gregory H. Wannier, *Statistical Physics*
Dover 1987 reprint of the 1966 edition, ISBN 0-486-65401-X

Do not forget to write your name and the homework number!

Each problem is worth 10 points.

Ch. 7 Thermodynamic Exploitation of the Second Law; Mass Transfer Problem

1. **UMD qualifier problem, August 2002: Boiling and freezing of water at high elevation.**

   (a) Near Earth, the pressure in a liquid or gas decreases as we move away from the earth. Show that, for a small vertical displacement dz, the change of pressure dP is
   $$dP = -\rho\, g\, dz \tag{1}$$
   where $\rho(z)$ is the mass density of the fluid.

   (b) Assuming that the air near Earth can be treated as a perfect gas in equilibrium with a constant temperature T, show that Eq. (1) yields the Boltzmann distribution for the variation of the number density $n(z)$ with the height z. (It is also called the barometric distribution.)

   (c) Derive the Maxwell relation
   $$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V . \tag{2}$$

   (d) Using Eq. (2) and/or other thermodynamic relationships, prove that water (ice) boils (melts) at lower (higher) temperatures when going from sea level to the top of a mountain.

2. **Problem 7.13, The Clausius-Clapeyron relation for an ideal gas.**

   Derive an analytical formula. Do not substitute the numbers for water.

3. **Problem 7.14, The Stefan-Boltzmann law for photons.**

   Use Eq. (7.07).

4. **Problem 7.15, Internal energy U for surface tension.**

   Generalize Eqs. (7.05) and (7.06) by adding the surface tension work and derive an analogue of Eq. (7.07) for $\partial U/\partial A$.

5. Problem 7.6, Prove an identity for $\partial H/\partial P$.

Use Eq. (7.12) and relationship between entropy and heat capacity (5.41).

6. Problem 7.7, Prove an identity for $\partial U/\partial P$.

Use Eq. (7.06) and express the differential dV in terms of dP and dT:

$$dV = \left(\frac{\partial V}{\partial P}\right)_T dP + \left(\frac{\partial V}{\partial T}\right)_P dT. \tag{3}$$

Then use cross-derivatives of J with respect to P and T.

7. Problem 7.8, Prove an identity for TdS.

Start from Eqs. (1.31) and (1.32) and express the differential dT in terms of dP and dV:

$$dT = \left(\frac{\partial T}{\partial P}\right)_V dP + \left(\frac{\partial T}{\partial V}\right)_P dV. \tag{4}$$

Use Eqs. (1.34), (1.35), and (1.09).

8. Problem 7.11, Prove an identity for the bulk modulus.

Use the definition (1.13b) of B and α.

Write

$$dV = \left(\frac{\partial V}{\partial P}\right)_T dP + \left(\frac{\partial V}{\partial T}\right)_P dT = \left(\frac{\partial V}{\partial P}\right)_S dP + \left(\frac{\partial V}{\partial S}\right)_P dS \tag{5}$$

and express

$$dS = \left(\frac{\partial S}{\partial P}\right)_T dP + \left(\frac{\partial S}{\partial T}\right)_P dT. \tag{6}$$

Substitute (6) into (5) and equate the coefficients in front of dP and dT.