Homework #5 — PHYS 603 — Spring 2007
Deadline: **Tuesday, April 24, 2007, in class**

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Textbook: Gregory H. Wannier, *Statistical Physics*
Dover 1987 reprint of the 1966 edition, ISBN 0-486-65401-X

Do not forget to write your name and the homework number!

Each problem is worth 10 points.

Ch. 5 Statistical Justification of the Second Law

1. **Problem 5.1, Adiabatic expansion of a cubic box.**

   Write down the quantum energy levels in a cubic box. Assuming that initially the energy levels are populated with a temperature T, find out how the temperature changes when the box is adiabatically expanded preserving its proportions. Show that a relationship between T and volume V is in agreement with Eq. (1.38) for monoatomic gas with $c_v = 3/2$.

   Problem 5.6, Asymmetric expansion of a rectangular box.

2. **Problem 5.2, Internal energy U and entropy S of the van der Waals gas.**

   Use the equation of state for the van der Waals gas given by Eq. (1.11).

3. **Problem 5.3, Change of entropy in mixing two ideal gases at different pressures.**

   The two gases initially have different pressures P_1 and P_2 and different volumes V_1 and V_2, but the same temperature T. Does the temperature change in the process of mixing?

   Use the expressions for entropy given in Eq. (8.54)

$$\begin{aligned} S &= N\,[\ln(eV/N) + c_v \ln T + c_v + \zeta], \qquad c_v = 3/2, \qquad c_p = 5/2, \\ S &= N\,[-\ln P + c_p \ln T + c_p + \zeta], \qquad \zeta = \frac{3}{2}\ln\left(\frac{2\pi m}{h^2}\right), \end{aligned} \tag{1}$$

   where we used the equation of state $P = (N/V)T$.

   Express the entropy change ΔS after mixing two gases in terms of the initial pressures P_1 and P_2 in a form similar to Eq. (5.63a). Using the arguments after this equation, show that $\Delta S \geq 0$, where the equality is achieved for $P_1 = P_2$.

   Answer part (b) of the problem as well.

4. **Problem 5.7, Adiabatic equation of state for diatomic molecules.**

   Internal energy for diatomic molecules has the form (4.48)

$$U = \frac{5}{2}NT. \tag{2}$$

   Here the contribution $(3/2)T$ comes from the 3 translational degrees of freedom, and the contribution $(2/2)T$ comes from 2 rotational degrees of freedom. Mechanical expansion of the box affects only the translational degrees of freedom.

5. **Problem 5.8, C_P/C_V for solids.**

6. **Problem 5.9, Entropy of a spin-1/2 magnet.**

7. **UMD qualifier problem, August 2000: A fiber with two types of segments.**

   Suppose that a very long straight fiber consists of N identical segments, each having two states. The state labeled α has energy ε_α and length l_α along the fiber, while the state β has energy ε_β and length l_β. The energy of each component is independent of the states of its neighbors. Thus, the total energy and length of the fiber for a particular arrangement of components is

$$E = N_\alpha \varepsilon_\alpha + N_\beta \varepsilon_\beta, \qquad L = N_\alpha l_\alpha + N_\beta l_\beta,$$

   where N_α and N_β are the numbers of segments in the states α and β; $N = N_\alpha + N_\beta$. The fiber is held under a constant tension force τ.

   (a) Recall that the Boltzmann entropy is $S = -k_B \sum_i P_i \ln P_i$, where the sum extends over all states occupied with probabilities $P_i = N_i/N$. Use the method of Lagrange multipliers to deduce the distribution of P_i that maximizes entropy subject to constraints that specify the ensemble averages $\langle E \rangle$ for energy and $\langle L \rangle$ for length. Identify the temperature and tension. This part is intended to be general, applicable to any one-dimensional system with constant temperature and tension.

   (b) For this particular system, find an explicit expression for the dependence of length upon temperature and tension. Sketch $\langle L \rangle$ as a function of the dimensionless parameter

$$\eta = \beta[(\varepsilon_\beta - \varepsilon_\alpha) - \tau(l_\beta - l_\alpha)].$$

   Make the sketch for η from $-\infty$ to $+\infty$. Explain the physical meaning of $\eta = -\infty, 0, +\infty$.

   (c) Demonstrate that at high temperatures one obtains a version of Hook's law by evaluating the isothermal elastic modulus

$$k = \left(\frac{\partial \tau}{\partial L}\right)_T.$$

   Explain the temperature dependence of k.

   (d) Suppose that the temperature is low enough so that $|\eta| \gg l_\beta/l_\alpha$. How does L depend upon τ? Describe the behavior of the fiber as τ is varied while the temperature is constant.