**Homework #3** — PHYS 603 — Spring 2007 Professor Victor Yakovenko
Deadline: **Thursday, April 5, 2007, in class** Office: 2115 Physics

Web page: http://www2.physics.umd.edu/~yakovenk/teaching/

Textbook: Gregory H. Wannier, *Statistical Physics*
Dover 1987 reprint of the 1966 edition, ISBN 0-486-65401-X

**Do not forget to write your name and the homework number!**

**Each problem is worth 10 points.**

# Ch. 3 Statistical Counting in Mechanics

1. **Problem 3.2, Phase space per quantum state for an oscillator.**

2. **Problem 3.4, Phase space per quantum state for a particle in a box.**

3. **Properties of the density matrix.**

   **(a)** Using Eq. (3.16), show that $\mathrm{Tr}\,\hat{\rho} = 1$.

   **(b)** Suppose Eq. (3.16) is written in the energy representation, where the indices $i$ and $j$ represent energy eigenstates. Then, the coefficients in Eq. (3.14) have the following time dependence: $a_j \propto e^{-iE_j t/\hbar}$. Show that, after averaging over a long time, the density matrix (3.16) becomes diagonal in the energy representation

   $$\langle i|\hat{\rho}|j\rangle = \delta_{ij} w_j, \tag{1}$$

   where $w_j = |a_j|^2$ are the probabilities to find the system in the energy states $j$. Show that the time-averaged density matrix corresponds to a mixed state and has the following form in the coordinate representation

   $$\rho(x, x') = \sum_j w_j \psi_j(x) \psi_j^*(x'), \tag{2}$$

   where $\psi_j(x)$ is the wave function of the energy eigenstate $j$.

4. **Two-particle density matrix.**

   **(a)** Consider two identical fermion particles of mass $m$ in a one-dimensional box of length $L$. Let us ignore the spin variable and consider "spinless fermions". Suppose the system is in the pure ground state with the wave function

   $$\psi(x_1, x_2) = \frac{1}{\sqrt{2}} \left[\psi_0(x_1)\psi_1(x_2) - \psi_1(x_1)\psi_0(x_2)\right], \tag{3}$$

   where $x_1$ and $x_2$ are the coordinates of the first and the second particles, and $\psi_0$ and $\psi_1$ are the normalized wave functions of the ground state and the first excited state in the box. The two-particle wave function (3) is antisymmetric with respect to $x_1$ and $x_2$, as required for identical fermions.

   Write down the density matrix in coordinate representation $\rho_{II}(x_1, x_2, x_1', x_2') = |\Psi\rangle\langle\Psi| = \psi(x_1, x_2)\, \psi^*(x_1', x_2')$ explicitly for the pure state (3).

**(b)** Take trace over the second particle and obtain the reduced one-particle density matrix $\rho_I(x_1,x_1') = \int dx_2\, \rho_{II}(x_1,x_2,x_1',x_2)$. Show that $\rho_I(x_1,x_1')$ represents a mixed state and obtain this function explicitly. Does $\rho_I(x_1,x_1')$ have the diagonal form (2)?